MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

DMA5201 – MATHEMATICAL TECHNIQUES 2

(TC1L)

10 JULY 2023
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **3** pages (**2** pages with **4** questions and **1** page of Appendix). Key formulae are given in the Appendix.

2. Answer **ALL** questions.

3. All necessary working steps must be shown in the Answer Booklet provided.

QUESTION 1 [15 Marks]

a) Find the derivatives of the following functions.

i. $y = \sin(x^3 + 2)$ (2 marks)

ii. $y = \frac{6x^2}{1+e^x}$ (2 marks)

b) Given $f(x) = (3x - 1)^4$ and $g(x) = 2 + \frac{1}{x}$.

i. Find $f'(x)$ and $g'(x)$. (2 marks)

ii. If $y = f(x) \cdot g(x)$. Show that $\frac{dy}{dx} = (3x - 1)^3 \left(24 + \frac{9}{x} + \frac{1}{x^2}\right)$. (2 marks)

c) Given $3y^5 + x^2y = 4x^3 - 10$. Use implicit differentiation to obtain $\frac{dy}{dx}$ in terms of x and y. (3.5 marks)

d) A cylindrical glass with radius 5 cm is being filled with water from a tap at a rate of 30 cm^3/sec. How fast is the water level in the glass rising? (3.5 marks)
[Volume of cylinder, $V = \pi r^2 h$]

QUESTION 2 [15 Marks]

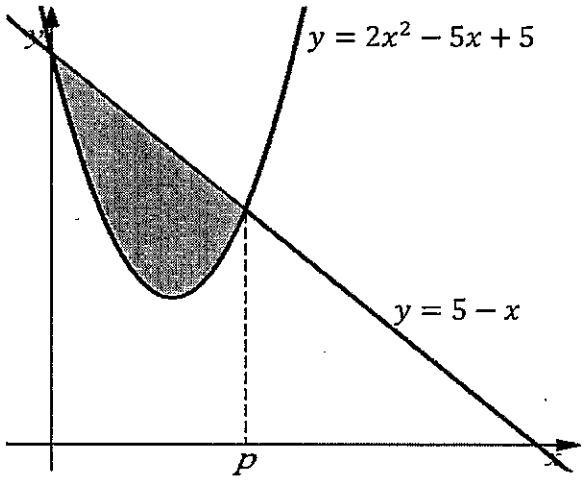
a) Find the following integrals:

i. $\int_0^1 5x^4 - e^x - 2\, dx$ (2 marks)

ii. $\int (2x^3 + 3x)(x^4 + 3x^2)^5\, dx$ [Hint: use Substitution Rule] (3.5 marks)

iii. $\int 10x \ln x\, dx$ [Hint: use Integration by Part] (3.5 marks)

b) The diagram below shows the curve of $y = 2x^2 - 5x + 5$ and the straight line of $y = 5 - x$.



i. Show that the value of p is 2. (2.5 marks)

ii. Find the area of the shaded region. (3.5 marks)

Continued...

QUESTION 3 **[10 Marks]**

a) Solve the first order separable equation $\frac{dy}{dx} = \frac{4x^3}{y^2}$ subject to the initial condition $y(0) = 6$. You may leave your answer in implicit form. (2.5 Marks)

b) You are told that e^{-3x} is an integrating factor (μ) for the first order linear equation:

$$\frac{dy}{dx} - 3y = 4xe^{3x}$$

Solve the equation and give your solution in explicit form.

(2 marks)

c) Given the following non-homogeneous differential equation $y'' - 4y' - 12y = 3x$.

i. Determine the complementary solution, y_c. (1.5 marks)

ii. Compute the particular solution, y_p. (3.5 marks)

iii. Hence, write down the general solution for the differential equation. (0.5 mark)

QUESTION 4 **[10 Marks]**

a) Given the difference between the 10th term and the 3rd term of an arithmetic sequence is $a_{10} - a_3 = 49$ and $a_3 = 19$. Find the first term, a_1 and common difference, d. (3 marks)

b) On 1st July, an IT engineer had 2310 microchips. From the following day onwards, he sold 210 microchips each day. Find the date where all microchips would be sold out. (2.5 marks)

c) Find the sum of the infinite geometric series, $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} + \ldots$ (1.5 marks)

d) Find the first four terms of the expansion $\left(x^3 + \frac{2}{x}\right)^9$. (3 marks)

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APPENDIX – KEY FORMULA

Differentiation Rules

1. $\frac{d}{dx}[x^n] = nx^{n-1}$
2. $\frac{d}{dx}[f(x)\cdot g(x)] = f(x)g'(x) + g(x)f'(x)$
3. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
4. $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$ or $\frac{d}{dx}[f(u)] = f'(u).\frac{du}{dx}$
5. $\frac{d}{dx}[\sin x] = \cos x$
6. $\frac{d}{dx}[\cos x] = -\sin x$
7. $\frac{d}{dx}[e^x] = e^x$
8. $\frac{d}{dx}[\ln x] = \frac{1}{x};\quad x > 0$

Integration Rules

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
2. $\int \frac{1}{x} dx = \ln x + C$
3. $\int e^x dx = e^x + C$
4. $\int u dv = uv - \int v du$
5. $\int \sin x\, dx = -\cos x + C$
6. $\int \cos x\, dx = \sin x + C$

Areas between Curve

$A = \int_a^b [f(x) - g(x)]\, dx$

Differential Equations

Linear Differential Equations

$\frac{dy}{dx} + p(x)y = q(x) \Rightarrow \mu y = \int \mu q(x)\, dx$, where $\mu = e^{\int p(x)dx}$

General Solutions to the Auxiliary Equation

2 Real & Unequal Roots ($b^2 - 4ac > 0$) $y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

Repeated Roots ($b^2 - 4ac = 0$) $y = c_1 e^{rx} + c_2 x e^{rx}$

2 Complex Roots ($b^2 - 4ac < 0$) $y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Arithmetic Sequence	*Geometric Sequence*
$a_n = a_1 + (n-1)d$ $S_n = \frac{n}{2}(a_1 + a_n)$	$a_n = a_1 r^{n-1}$, $S_n = \frac{a_1(1-r^n)}{1-r}$ $S_\infty = \frac{a_1}{1-r}, \lvert r\rvert < 1$

Binomial Theorem

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k;\quad n \geq 1$

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